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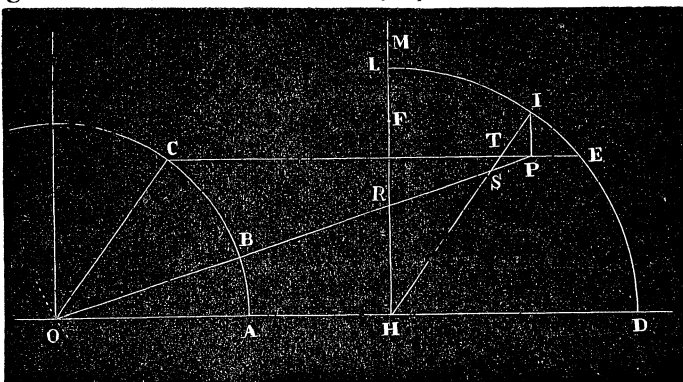
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TRISECTION OF AN ANGLE.

BY DR. WILLIAM HILLHOUSE, NEWHAVEN, CONN.

DRAW a straight line OD , and on it erect a perpendicular HM . Take

any point F , in HM and from F with twice the distance FH , as radius, describe an arc cutting the line OD in O . With O as center and radius FH , (or half OF), de-



scribe the arc ABC . Again, from the point O , lay off, on the line OH , three times OA , to the point D , and with H as center and radius HD , describe the arc DEL .

To trisect any angle from 0 to 90° , say AOC , draw OC , making AOC = the given angle; through C draw CE parallel to OD and draw the radius HI parallel to OC . Draw IP perpendicular to CE and join OP , cutting the arc AC in B , then will the angle AOB equal one-third of the angle AOC .

[The foregoing construction is somewhat remarkable in that it gives accurate results at the two limits of an arc of 90° ; that is, the trisection is exact for an arc of 90° and also for an infinitely small arc, and is a close approximation for all intervening arcs; so close, indeed, that a few trial constructions would be likely to induce a casual observer to believe that the trisection is exact:—That it is not, may be proved as follows:

Assume that the trisection, as represented in the figure, is exact. Then, from a consideration of the triangles OHS and OHR , we can easily prove that $OR = 2HS$.

From the similar triangles OHS and PTS we find

$$OR = \frac{s \sqrt{(rv + s)^2 + 2v(rv + s) + 1}}{rv + v + s}$$

and

$$HS = \frac{s}{rv + s},$$

where v = cosine of the angle to be trisected, $s = OH = \sqrt{3}$ and $r = TI = 2 - \sqrt{3}$. If the construction is exact we shall therefore have

$$\frac{s\sqrt{[(rv + s)^2 + 2v(rv + s) + 1]}}{rv + v + s} = \frac{2s}{rv + s}.$$

Whence $\sqrt{[(rv + s)^2 + 2v(rv + s) + 1]} = 2(rv + v + s)$.

Because v cannot be eliminated from this equation therefore the trisection cannot, in general, be exact; otherwise we would have an equation of the 4th degree with an infinite number of roots.

Moreover, as the above equation is verified by substituting for v its value at either limit, and as it has two negative roots, it follows that the foregoing construction will not trisect *any* finite angle except one of 90° .—Ed.]

SOLUTION OF A PROBLEM IN PROBABILITIES.

BY THE EDITOR.

IF four bricks are placed on each other at random, with their longest axes horizontal and in the same vertical plane, determine the probability that the pile will stand.*

Let the bricks, in the order in which they are placed on the pile, be represented respectively by (1), (2), (3), (4).

Denote the length of a brick by a , and the horizontal distance between the centers of (1) and (2), (2) and (3), (3) and (4), respectively, by x , y , z .

Then, if x be not greater than $\frac{1}{6}a$, and if y is estimated in the same direction from the center of (1) as x is, and is taken not greater than $\frac{1}{4}a$, the center of (4) may take any position on (3) and the pile will stand. This 1st case therefore gives, for the number of stable positions of the pile,

$$\frac{1}{6}a \times \frac{1}{4}a \times a = u_1 = \frac{1}{24}a^3.$$

Also, while x is restricted to $\frac{1}{6}a$, y may take any value between $\frac{1}{4}a$ and $\frac{1}{2}a$, but the *range* of the center of (4), for stable positions of the pile, will now be $\frac{1}{2}(3a - 4y)$; hence the number of stable positions in this case will be

$$\frac{1}{6}a \int_{\frac{1}{4}a}^{\frac{1}{2}a} \frac{1}{2}(3a - 4y)dy = \frac{1}{6}a \left[\frac{3}{2}ay - y^2 \right]_{\frac{1}{4}a}^{\frac{1}{2}a} = u_2 = \frac{1}{8}a^3.$$

In like manner we may find that for values of x between $\frac{1}{6}a$ and $\frac{1}{4}a$, y may have any value between 0 and $\frac{1}{2}(a - 3x)$, and z any value from $-\frac{1}{2}a$ to $+\frac{1}{2}a$; hence the number of stable positions will be

*This problem was, I believe, first proposed by Artemas Martin, in the Schoolday Visitor for Dec. 1871. No solution was published in that periodical, nor elsewhere so far as I know, though two other solutions of the question have been made, independently; one by E. B. Seitz and one by Henry Heaton.